Further Pure Mathematics 1

The factor theorem

The factor theorem states that:

$$f(a) = 0 \Leftrightarrow (x - a) \text{ is a factor of } f(x)$$

For example, suppose the cubic polynomial $f(x)$ is defined by

$f(x) = x^3 - 2x^2 - x + 2$

You can work out the value of $f(x)$ for different values of x.

$$f(1) = 1^3 - 2\times 1^2 - 1 + 2 = 1 - 2 - 1 + 2 = 0$$
$$f(2) = 2^3 - 2\times 2^2 - 2 + 2 = 8 - 8 - 2 + 2 = 0$$
$$f(3) = 3^3 - 2\times 3^2 - 3 + 2 = 27 - 18 - 3 + 2 = 8$$
$$f(-1) = (-1)^3 - 2(-1)^2 + 1 + 2 = -1 - 2 + 1 + 2 = 0$$
$$f(-2) = (-2)^3 - 2(-2)^2 + 2 + 2 = -8 - 8 + 2 + 2 = -12$$
$$f(-3) = (-3)^3 - 2(-3)^2 + 3 + 2 = -27 - 18 + 3 + 2 = -41$$

You can see that the polynomial is zero when x = 1, 2 or -1. (In fact these are the only values of x for which the polynomial is zero.)

The factor theorem tells us that $(x - 1)$, $(x - 2)$ and $(x + 1)$ are factors of $f(x)$.
Hence $f(x) = (x - 1)(x - 2)(x + 1)$.

Usually, if you need to factorise a cubic equation, you find one factor by trial and error, and then factorise the cubic into a linear factor and a quadratic factor. This is shown below.

Example
Solve the equation $2x^3 - 5x^2 - 4x + 3 = 0$

Solution
$f(x) = 2x^3 - 5x^2 - 4x + 3$
$f(1) = 2\times 1^3 - 5\times 1^2 - 4\times 1 + 3 = 2 - 5 - 4 + 3 = -4$
$f(2) = 2\times 2^3 - 5\times 2^2 - 4\times 2 + 3 = 16 - 20 - 8 + 3 = -9$
$f(3) = 2\times 3^3 - 5\times 3^2 - 4\times 3 + 3 = 54 - 45 - 12 + 3 = 0$
so $(x - 3)$ is a factor.

Try some different values for x until you find one which makes the polynomial zero

$2x^3 - 5x^2 - 4x + 3 = 0$
$(x - 3)(2x^2 + x - 1) = 0$
$(x - 3)(2x - 1)(x + 1) = 0$

The cubic can be factorised into the linear factor $(x - 3)$, and a quadratic factor. **PowerPoint presentation** showing how this is done.

Now the quadratic can be factorised.

The solution is $x = 3, \frac{1}{2}$, or -1

For more help, look at C1 chapter 3.